Chapter 2 Curriculum Framework

The curriculum framework for Mathematics embodies the key knowledge, skills, values and attitudes that students are to develop at the senior secondary level. It forms the basis on which schools and teachers can plan their school-based curricula, and design appropriate learning, teaching and assessment activities.

2.1 Design Principles

The following principles are used in designing the curriculum:

(a) Building on knowledge developed at the basic education level

To ensure that the curricula at different levels of schooling are coherent, the development of the Mathematics Curriculum (S4 – 6) is built on the knowledge, skills, values and attitude acquired through the Mathematics Curriculum for basic education from Primary 1 to Secondary 3.

(b) Providing a balanced, flexible and diversified curriculum

With the implementation of the senior secondary academic structure in Hong Kong, a wider range of students will gain access to Mathematics at the senior secondary level than in the past. The Mathematics Curriculum (S4 – 6) offers a Compulsory Part and an Extended Part. The Compulsory Part is a foundation for all students and provides mathematical concepts, skills and knowledge which are necessary for students' different career pathways. The Extended Part embraces two optional modules to provide add-on mathematical knowledge to suit the individual needs of students who would like to learn more mathematics and in a greater depth. The curriculum thus provides flexibility for teachers to:

- offer a choice of courses within the curriculum to meet students' individual needs, e.g. Compulsory Part, Compulsory Part with Module 1 (Calculus and Statistics) or Compulsory Part with Module 2 (Algebra and Calculus);
- organise the teaching sequence to meet individual situations; and
- make adaptations to the content.

(c) Catering for learner diversity

The curriculum provides opportunities for organising a variety of student activities to cater for learner diversity. The learning unit "Inquiry and Investigation" in the curriculum allows teachers to plan different learning activities for individual students. To further assist teachers to adapt the curriculum to suit the needs of individual groups of students, the content in the Compulsory Part is categorised into Foundation Topics and Non-foundation Topics. The Foundation Topics constitute a set of essential concepts and knowledge which **all** students should strive to learn. Teachers can judge for themselves the suitability and relevance of the content from the Non-foundation Topics for their own students. The Extended Part comprises two modules with different orientations. Students who are more able in mathematics, more mathematically oriented or need more mathematical knowledge and skills to prepare for their future studies and careers may choose to study a module from the Extended Part. Module 1 (Calculus and Statistics) focuses more on mathematical applications, whereas Module 2 (Algebra and Calculus) places more emphasis on mathematical concepts and knowledge. Students who would like to learn more mathematics may choose the module which best suits their interests and needs.

(d) Achieving a balance between breadth and depth

The curriculum covers the important and relevant contents for senior secondary students, based on the views of mathematicians, professionals in Mathematics Education and overseas Mathematics curricula at the same level. The breadth and depth of treatment in the Extended Part are intended to provide more opportunities for intellectually rigorous study in the subject.

(e) Achieving a balance between theoretical and applied learning

An equal emphasis is given on theories and applications in both real-life and mathematical contexts to help students construct their knowledge and skills in Mathematics. The historical development of selected mathematical topics is also included to promote students' understanding of how mathematics knowledge has evolved and been refined in the past.

(f) Fostering lifelong learning skills

Knowledge is expanding at an ever faster pace and new challenges are continually posed by rapid developments in technology. It is important for our students to learn how to learn, think critically, analyse and solve problems, and communicate with others effectively so that they can confront current and future challenges. The curriculum provides a suitable context for developing such abilities.

(g) Promoting positive values and attitudes to learning

Positive values and attitudes to learning, which are important in learning mathematics, permeate the Mathematics Curriculum (S4 – 6). In particular, the unit "Inquiry and Investigation" helps to develop in students an interest in learning mathematics, keenness to participate in mathematics activities, and sensitivity and confidence in applying mathematics in daily life. It also helps to foster open-mindedness and independent thinking.

2.2 The Mathematics Education Key Learning Area Curriculum Framework

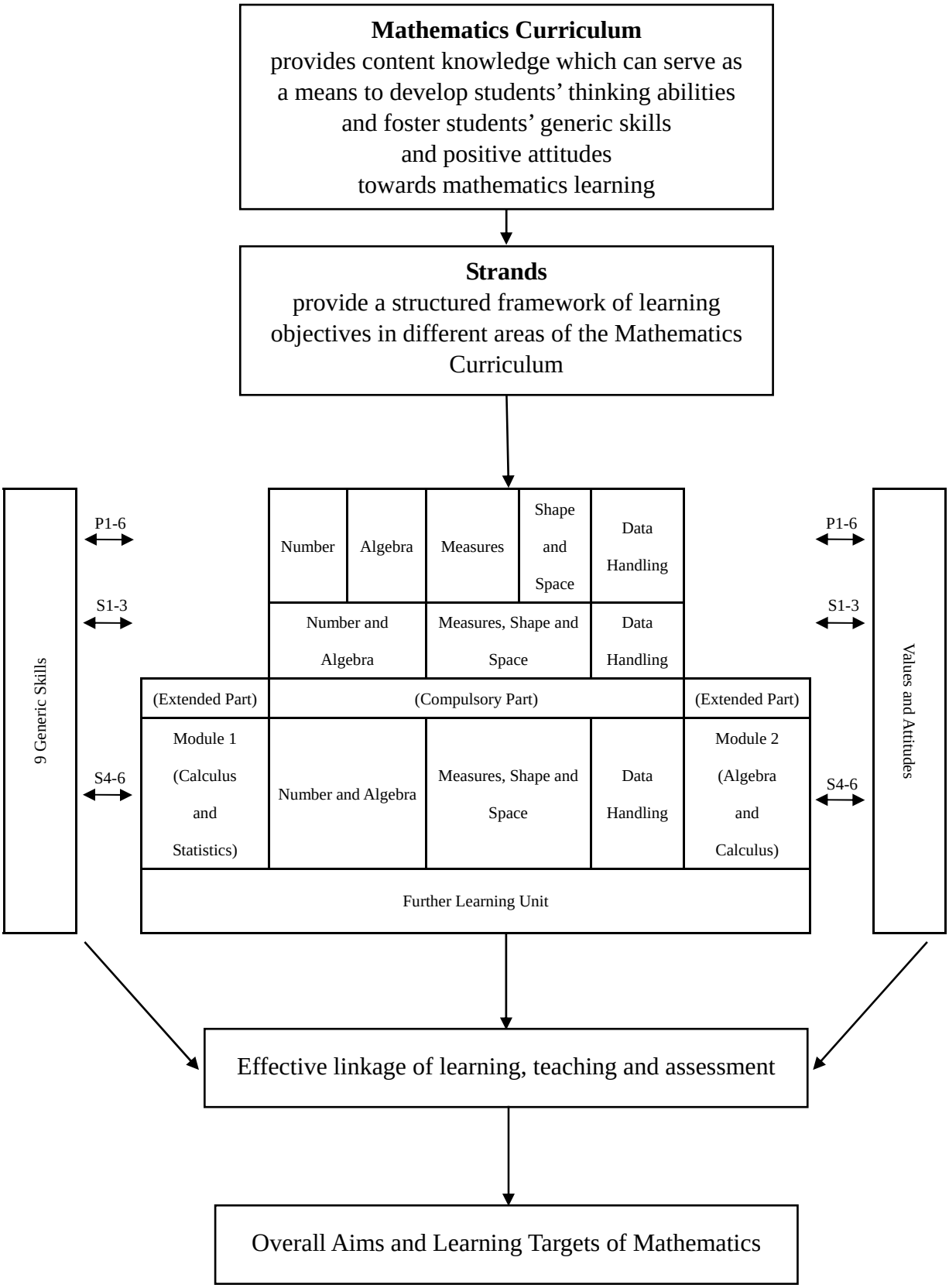
The curriculum framework for Mathematics Education is the overall structure for organising learning and teaching activities for the subject of Mathematics. The framework comprises a set of interlocking components, including:

- subject knowledge and skills, which are expressed in the form of learning targets and learning objectives within strands;
- generic skills; and
- positive values and attitudes.

The framework sets out what students should know, value and be able to do at various stages of schooling from Primary 1 to Secondary 6. It provides schools and teachers with the flexibility to adapt the Mathematics Curriculum to meet their varied needs.

A diagrammatic representation highlighting the major components of the Mathematics Curriculum framework is provided on the following page.

Diagrammatic Representation of the Framework of the Mathematics Curriculum



2.2.1 Strands

Strands are categories of mathematical knowledge and concepts for organising the curriculum. Their main function is to organise mathematical content for the purpose of developing knowledge, skills, values and attitudes as a holistic process. The contents of the Mathematics Curriculum are organized into five strands at the primary level and three strands at the secondary level. In particular, the Compulsory Part of the Mathematics Curriculum (S4 – 6) comprises three strands, namely "Number and Algebra", "Measures, Shape and Space" and "Data Handling". As the contents of the Extended Part are interwoven, they are not categorised into strands.

2.2.2 Generic Skills

Generic skills can be seen as both process skills and learning outcomes in the Mathematics Education Key Learning Area. They are essential for enabling learners to learn how to learn. Nine generic skills have been identified: collaboration skills, communication skills, creativity, critical thinking skills, information technology skills, numeracy skills, problem-solving skills, self-management skills and study skills.

It should be noted that generic skills are not something to be added on to the learning and teaching of mathematical concepts, but should be embedded within them. They serve as a means to develop the acquisition and mastery of mathematical knowledge and concepts. An emphasis on communication skills, creativity and critical thinking skills in the context of mathematical activities will help to strengthen students' ability to achieve the overall learning targets of the curriculum. Daily-life applications, further applications of mathematics, inquiry and investigation are emphasised.

2.2.3 Values and Attitudes

Besides knowledge and skills, the development of positive values and attitudes is also important in Mathematics Education. Values and attitudes such as responsibility, commitment and open-mindedness are necessary for developing goals in life and learning. The inculcation of such positive values/attitudes through appropriate learning and teaching strategies can enhance learning, and this in turn will reinforce their development in students as part of character formation. Positive values and attitudes permeate the Mathematics

Curriculum (S4 – 6) and have been incorporated into its learning objectives, so that students can:

- develop **interest** in learning mathematics;
- show **keenness** to participate in mathematical activities;
- develop **sensitivity** towards the importance of mathematics in daily life;
- show **confidence** in applying mathematical knowledge in daily life, by clarifying one's argument and challenging others' statements;
- share ideas and experience and work **cooperatively** with others in accomplishing mathematical tasks/activities and solving mathematical problems;
- understand and take up **responsibilities**;
- be **open-minded**, willing to listen to others in the discussion of mathematical problems, respect others' opinions, and value and appreciate others' contributions;
- **think independently** in solving mathematical problems;
- be **persistent** in solving mathematical problems; and
- **appreciate** the precise, aesthetic and cultural aspects of mathematics and the role of mathematics in human affairs.

These values and attitudes can be fostered through the learning of mathematical contents. Teachers can help students cultivate them through planned learning activities.

2.3 Aims of Senior Secondary Mathematics Curriculum

The Mathematics Curriculum (S4 – 6) is a continuation of the Mathematics Curriculum (S1 – 3). It aims to:

(a) further develop students' mathematical knowledge, skills and concepts;

(b) provide students with mathematical tools for their personal development and future career pathways;

(c) provide a foundation for students who may further their studies in Mathematics or related areas;

(d) develop in students the generic skills, and in particular, the capability to use mathematics to solve problems, reason and communicate;

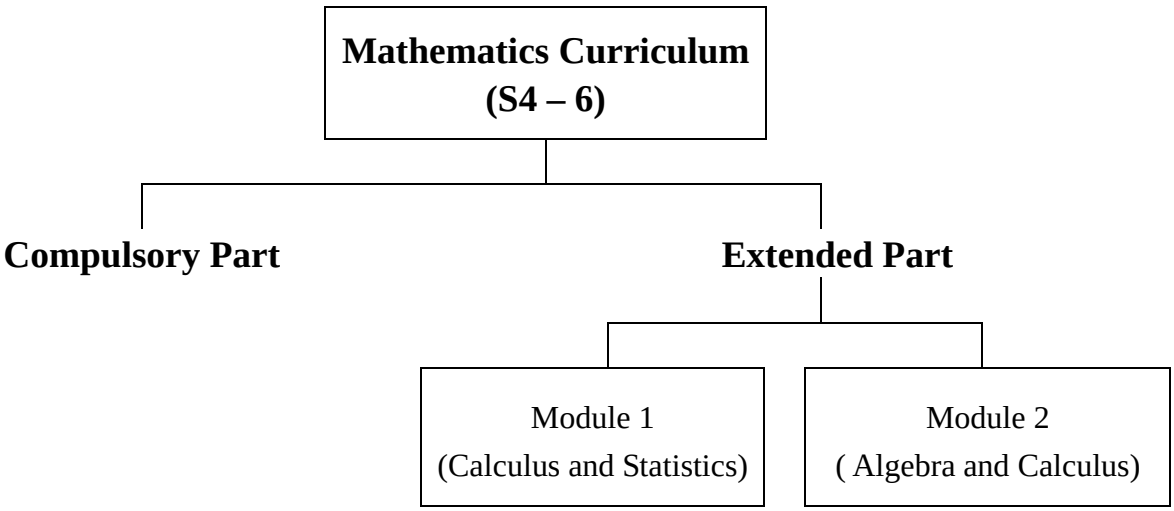
(e) develop in students interest in and positive attitudes towards mathematics learning;

(f) develop students' competence and confidence in dealing with mathematics needed in life; and

(g) help students to fulfil their potential in mathematics.

2.4 Framework of Senior Secondary Mathematics Curriculum

The structure of the Mathematics Curriculum (S4 – 6) can be represented diagrammatically as follows:



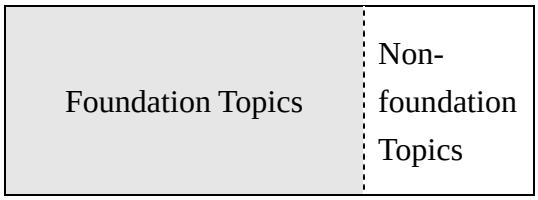
[*Note: Students may take the Compulsory Part only, the Compulsory Part with Module 1 (Calculus and Statistics) or the Compulsory Part with Module 2 (Algebra and Calculus). Students are only allowed to take at most one module from the Extended Part.*]

To cater for students who have different needs, interests and orientations, the curriculum comprises a Compulsory Part and an Extended Part. **All** students must study the Compulsory Part.

The Extended Part has two optional modules, namely Module 1 (Calculus and Statistics) and Module 2 (Algebra and Calculus). The inclusion of the Extended Part is designed to provide more flexibility and diversity in the curriculum. The two modules in the Extended Part provide additional mathematics knowledge to the Compulsory Part. Students, based on their individual needs and interests, are encouraged to take at most one of the two modules.

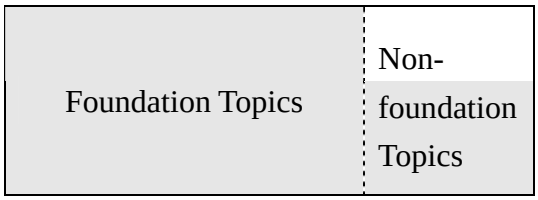
The following diagrams show the different ways in which students can progress:

(1) Students who study only the Foundation Topics in the Compulsory Part



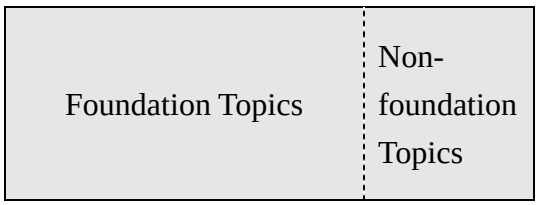
Compulsory Part

(2) Students who study the Foundation Topics and some Non-foundation Topics in the Compulsory Part



Compulsory Part

(3) Students who study all topics in the Compulsory Part



Compulsory Part

(4) Students who study the Compulsory Part with Module 1 (Calculus and Statistics)

Compulsory Part	**Module 1 (Calculus and Statistics)**

(5) Students who study the Compulsory Part with Module 2 (Algebra and Calculus)

Compulsory Part	**Module 2 (Algebra and Calculus)**

As a core subject, the Mathematics Curriculum (S4 – 6) accounts for up to 15% (approximately 405 hours) of the total lesson time available in the senior secondary curriculum. The suggested time allocations for the Compulsory Part and the Extended Part are as follows:

	Lesson time (Approximate number of hours)
Compulsory Part	10% – 12.5% (270 hours – 338 hours)
Compulsory Part with a module	15% (405 hours)

2.5 Compulsory Part

The principles of curriculum design of the Compulsory Part comply with those of the Mathematics Curriculum (S4 – 6) as a whole, but have two distinguishing features.

First, the Compulsory Part serves as a foundation for all students and at the same time provides flexibility to cater for the diverse needs of individual students. Its contents are

categorised into Foundation Topics and Non-foundation Topics. The Foundation Topics constitute a coherent set of essential concepts and knowledge while the Non-foundation Topics cover a wider range of content.

Second, the topics in the Compulsory Part emphasise the relevance of mathematics to various human activities. Students are expected to engage in different activities to foster their awareness of the worldwide use of mathematical terminology, notation and strategies to solve problems. Also, to enable students to recognise and appreciate the interconnection between the different parts of mathematics they have learnt at both the junior and senior secondary levels, a "Further Applications" learning unit is incorporated into the Compulsory Part.

The learning objectives of the Compulsory Part foster students' understanding of the development of mathematical knowledge and skills and their applications in the solution of various problems, including real-life ones. In addition, learning units such as "Uses and Abuses of Statistics", "Permutation and Combination" and "Further Applications" are included for students to use the mathematics learnt at junior and senior secondary levels to understand and assess more sophisticated scenarios critically.

2.5.1 Organisation of the Compulsory Part

The most significant aspects of learning and teaching in each strand of the Compulsory Part are organised into a hierarchy from Learning Targets to specific Learning Objectives. Learning Targets set out the aims and direction for learning and teaching and, under these, Learning Objectives are identified to spell out specifically what students need to learn. In the curriculum, Learning Objectives are presented and grouped under different Learning Units.

The three strands in the Compulsory Part are "Number and Algebra", "Measures, Shape and Space" and "Data Handling". In addition, the "Further Learning Unit" is designed to integrate and apply knowledge and skills learned in the strands to solve problems in real-life as well as in mathematical contexts.

2.5.2 Learning Targets of the Compulsory Part

An overview of the learning targets of the three strands in the Compulsory Part is provided on the following page.

Learning Targets in the Compulsory Part

Number and Algebra Strand	Measures, Shape and Space Strand	Data Handling Strand
Students are expected to:		
• extend the concepts of numbers to complex numbers; • investigate and describe relationships between quantities using algebraic symbols; • generalise and describe patterns in sequences of numbers using algebraic symbols, and apply the results to solve problems; • interpret more complex algebraic relations from numerical, symbolic and graphical perspectives; • manipulate more complex algebraic expressions and relations, and apply the knowledge and skills to formulate and solve real-life problems and justify the validity of the results obtained; and • apply the knowledge and skills in the Number and Algebra strand to generalise, describe and communicate mathematical ideas and solve further problems in other strands.	• use inductive and deductive approaches to study the properties of 2-dimensional shapes; • formulate and write geometric proofs involving 2-dimensional shapes with appropriate symbols, terminology and reasons; • inquire into and describe geometric knowledge in 2-dimensional space using algebraic relations and apply this knowledge in solving related problems; • inquire and describe geometric knowledge in 2-dimensional space and 3-dimensional space using trigonometric functions and apply the knowledge in solving related problems; and • interconnect the knowledge and skills in the Measures, Shape and Space strand and other strands, and apply them to formulate and solve 2-dimensional and 3-dimensional problems using various strategies.	• understand the measures of dispersion; • select and use the measures of central tendency and dispersion to compare data sets; • investigate and judge the validity of arguments derived from data sets; • acquire basic techniques in counting; • formulate and solve further probability problems by applying simple laws; and • integrate the knowledge in statistics and probability to solve real-life problems.

2.5.3 Foundation Topics and Non-foundation Topics in the Compulsory Part

To cater for the needs of individual students, the contents of the Compulsory Part are categorised into Foundation Topics and Non-foundation Topics. The Foundation Topics of the Compulsory Part, together with the Foundation Part of the Mathematics Curriculum (S1 – 3), constitute a coherent set of essential concepts and knowledge. The Foundation Topics, which all students should strive to learn, are selected in accordance with the following principles in mind:

- to include basic concepts and knowledge necessary for the learning content in the Compulsory Part and for simple applications in real-life situations; and
- to cover topics from different areas to enable students to develop a coherent body of knowledge and to experience mathematics from an all-around perspective.

There are also contents beyond those in the Foundation Topics in terms of depth and breadth. They are identified as Non-foundation Topics and cover a wider range of contents, to provide students who study only the Compulsory Part with a foundation for their future studies and career development. Teachers can judge for themselves the suitability and relevance of the Non-foundation Topics for their own students.

The contents of Module 1 and Module 2 are built upon the study of the Foundation and Non-foundation Topics in the Compulsory Part. It is advisable for students to study both the Foundation Topics and Non-foundation Topics in the Compulsory Part if they study either one of the modules from the Extended Part.

2.5.4 Learning Objectives of the Compulsory Part

The time allocated to the Compulsory Part ranges from 10% to 12.5% of the total lesson time (approximately 270 hours to 338 hours), subject to the different pathways, orientations and learning speeds of students.

To aid teachers in their planning and adaptation, a suggested lesson time in hours is given against each learning unit in the following table. The learning objectives of the Non-foundation Topics are <u>underlined</u> for teachers' reference.

The Learning Objectives of the Compulsory Part

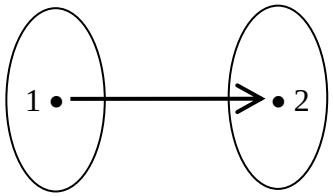
Notes:

1. *Learning units are grouped under three strands ("Number and Algebra", "Measures, Shape and Space" and "Data Handling") and a Further Learning Unit.*
2. *Related learning objectives are grouped under the same learning unit.*
3. *The learning objectives underlined are the Non-foundation Topics.*
4. *The notes in the "Remarks" column of the table may be considered as supplementary information about the learning objectives.*
5. *To aid teachers in judging how far to take a given topic, a suggested lesson time in hours is given against each learning unit. However, the lesson time assigned is for their reference only. Teachers may adjust the lesson time to meet their individual needs.*
6. *Schools may allocate up to 338 hours (i.e. 12.5% of the total lesson time) to those students who need more time for learning.*

Learning Unit	Learning Objective	Time	Remarks
Number and Algebra Strand			
1. Quadratic equations in one unknown	1.1 solve quadratic equations by the factor method	19	
	1.2 form quadratic equations from given roots		The given roots are confined to real numbers.
	1.3 solve the equation $ax^2 + bx + c = 0$ by plotting the graph of the parabola $y = ax^2 + bx + c$ and reading the x-intercepts		

Learning Unit	Learning Objective	Time	Remarks
	1.4 solve quadratic equations by the quadratic formula		The following are **not** required for students taking only the Foundation Topics: • expressing nonreal roots in the form $a \pm bi$ • simplifying expressions involving surds such as $2 \pm \sqrt{48}$
	1.5 understand the relations between the discriminant of a quadratic equation and the nature of its roots		When $\Delta < 0$, students have to point out that "the equation has **no real roots**" or "the equation has **two nonreal roots**" as they are expected to recognise the existence of complex numbers in Learning Objective 1.8.

Learning Unit	Learning Objective	Time	Remarks
	1.6 solve problems involving quadratic equations		Teachers should select the problems related to students' experiences. Problems involving complicated equations such as $\frac{6}{x}+\frac{6}{x-1}=5$ are required only in the Non-foundation Topics and tackled in Learning Objective 5.4.
	1.7 <u>understand the relations between the roots and coefficients and form quadratic equations using these relations</u>		The relations between the roots and coefficients include: • $\alpha+\beta=-\frac{b}{a}$ and $\alpha\beta=\frac{c}{a}$, where α and β are the roots of the equation $ax^2+bx+c=0$ and $a\neq 0$.
	1.8 appreciate the development of the number systems including the system of complex numbers		The topics such as the hierarchy of the number systems and the conversion between recurring decimals and fractions may be discussed.

Learning Unit	Learning Objective	Time	Remarks
	1.9 <u>perform addition, subtraction, multiplication and division of complex numbers</u>		Complex numbers are confined to the form $a \pm bi$. Note: The coefficients of quadratic equations are confined to real numbers.
2. Functions and graphs	2.1 recognise the intuitive concepts of functions, domains and co-domains, independent and dependent variables 2.2 recognise the notation of functions and use tabular, algebraic and graphical methods to represent functions 2.3 understand the features of the graphs of quadratic functions	10	Finding the domain of a function is required but need **not** be stressed. Representations like  are also accepted. The features of the graphs of quadratic functions include: • the vertex • the axis of symmetry • the direction of opening • relations with the axes Students are expected to find the maximum

Learning Unit	Learning Objective	Time	Remarks
	2.4 <u>find the maximum and minimum values of quadratic functions by the algebraic method</u>		and minimum values of quadratic functions by the graphical method. Students are expected to solve problems relating to maximum and minimum values of quadratic functions.
3. Exponential and logarithmic functions	3.1 <u>understand the definitions of rational indices</u> 3.2 <u>understand the laws of rational indices</u>	16	The definitions include $\sqrt[n]{a}$, $a^{\frac{1}{n}}$ and $a^{\frac{m}{n}}$. Students are also expected to evaluate expressions such as $\sqrt[3]{-8}$. The laws of rational indices include: • $a^p a^q = a^{p+q}$ • $\frac{a^p}{a^q} = a^{p-q}$ • $(a^p)^q = a^{pq}$ • $a^p b^p = (ab)^p$ • $\frac{a^p}{b^p} = \left(\frac{a}{b}\right)^p$

Learning Unit	Learning Objective	Time	Remarks
	3.3 understand the definition and properties of logarithms (including the change of base)		The properties of logarithms include: • $\log_a 1 = 0$ • $\log_a a = 1$ • $\log_a MN = \log_a M + \log_a N$ • $\log_a \frac{M}{N} = \log_a M - \log_a N$ • $\log_a M^k = k \log_a M$ • $\log_b N = \frac{\log_a N}{\log_a b}$
	3.4 understand the properties of exponential functions and logarithmic functions and recognise the features of their graphs		The following properties and features are included: • the domains of the functions • the function $f(x) = a^x$ increases (decreases) as x increases for $a > 1$ $(0 < a < 1)$ • $y = a^x$ is symmetric to $y = \log_a x$ about $y = x$ • the intercepts with the axes • the rate of increasing/the rate of decreasing (by direct inspection)

Learning Unit	Learning Objective	Time	Remarks
	3.5 <u>solve exponential equations and logarithmic equations</u>		Equations which can be transformed into quadratic equations such as $4^x - 3 \cdot 2^x - 4 = 0$ or $\log(x - 22) + \log(x + 26) = 2$ are tackled in Learning Objective 5.3.
	3.6 <u>appreciate the applications of logarithms in real-life situations</u>		The applications such as measuring earthquake intensity in the Richter Scale and sound intensity level in decibels may be discussed.
	3.7 <u>appreciate the development of the concepts of logarithms</u>		The topics such as the historical development of the concepts of logarithms and its applications to the design of some past calculation tools such as slide rules and the logarithmic table may be discussed.
4. More about polynomials	4.1 perform division of polynomials	14	Methods other than long division are also accepted.
	4.2 understand the remainder theorem		
	4.3 understand the factor theorem		

Learning Unit	Learning Objective	Time	Remarks
	4.4 understand the concepts of the greatest common divisor and the least common multiple of polynomials 4.5 perform addition, subtraction, multiplication and division of rational functions		The terms “H.C.F.” , “gcd”, etc. can be used. Computation of rational functions with more than two variables is **not** required.
5. More about equations	5.1 use the graphical method to solve simultaneous equations in two unknowns, one linear and one quadratic in the form $y = ax^2 + bx + c$ 5.2 use the algebraic method to solve simultaneous equations in two unknowns, one linear and one quadratic 5.3 solve equations (including fractional equations, exponential equations, logarithmic equations and trigonometric equations) which can be transformed into quadratic equations 5.4 solve problems involving equations which can be transformed into quadratic equations	10	Solutions for trigonometric equations are confined to the interval from 0° to 360° . Teachers should select the problems related to students’ experience.
6. Variations	6.1 understand direct variations (direct proportions) and inverse variations (inverse proportions), and their applications to solving real-life problems	9	

Learning Unit	Learning Objective	Time	Remarks
	6.2 understand the graphs of direct and inverse variations 6.3 understand joint and partial variations, and their applications to solving real-life problems		
7. Arithmetic and geometric sequences and their summations	7.1 understand the concept and the properties of arithmetic sequences 7.2 understand the general term of an arithmetic sequence 7.3 understand the concept and the properties of geometric sequences 7.4 understand the general term of a geometric sequence	17	The properties of arithmetic sequences include: • $T_n = ½ (T_{n-1} + T_{n+1})$ • if $T_1 , T_2 , T_3 , \ldots$ is an arithmetic sequence, then $k T_1 + a$, $k T_2 + a$, $k T_3 + a$, … is also an arithmetic sequence The properties of geometric sequences include: • $T_n^2 = T_{n-1} \times T_{n+1}$ • if $T_1 , T_2 , T_3 , \ldots$ is a geometric sequence, then $k T_1$, $k T_2$, $k T_3$, …is also a geometric sequence

Learning Unit	Learning Objective	Time	Remarks
	7.5 understand the general formulae of the sum to a finite number of terms of an arithmetic sequence and a geometric sequence and use the formulae to solve related problems 7.6 explore the general formulae of the sum to infinity for certain geometric sequences and use the formulae to solve related problems 7.7 solve related real-life problems		Example: geometrical problems relating to the sum of arithmetic or geometric sequences. Example: geometrical problems relating to infinite sum of the geometric sequences. Examples: problems about interest, growth or depreciation.
8. Inequalities and linear programming	8.1 solve compound linear inequalities in one unknown 8.2 solve quadratic inequalities in one unknown by the graphical method 8.3 solve quadratic inequalities in one unknown by the algebraic method 8.4 represent the graphs of linear inequalities in two unknowns on a plane	16	Compound inequalities involving logical connectives “and” or “or” are required.

Learning Unit	Learning Objective	Time	Remarks
	8.5 solve systems of linear inequalities in two unknowns 8.6 solve linear programming problems		
9. More about graphs of functions	9.1 sketch and compare graphs of various types of functions including constant, linear, quadratic, trigonometric, exponential and logarithmic functions 9.2 solve the equation $f(x) = k$ using the graph of $y = f(x)$ 9.3 solve the inequalities $f(x) > k$, $f(x) < k$, $f(x) \geq k$ and $f(x) \leq k$ using the graph of $y = f(x)$ 9.4 understand the transformations of the function $f(x)$ including $f(x) + k$, $f(x + k)$, $k f(x)$ and $f(kx)$ from tabular, symbolic and graphical perspectives	11	Comparison includes domains, existence of maximum or minimum values, symmetry and periodicity.
Measures, Shape and Space Strand			
10. Basic properties of circles	10.1 understand the properties of chords and arcs of a circle	23	The properties of chords and arcs of a circle include: • the chords of equal arcs are equal • equal chords cut off equal arcs

Learning Unit	Learning Objective	Time	Remarks
			• the perpendicular from the centre to a chord bisects the chord • the straight line joining the centre and the mid-point of a chord which is not a diameter is perpendicular to the chord • the perpendicular bisector of a chord passes through the centre • equal chords are equidistant from the centre • chords equidistant from the centre are equal Students are expected to understand why there is one and only one circle passing through given three non-collinear points. Note: the property that the arcs are proportional to their corresponding angles at the centre should be discussed at Key Stage 3 when the formula for calculating arc lengths is being explicated.

Learning Unit	Learning Objective	Time	Remarks
	10.2 understand the angle properties of a circle		Angle properties of a circle include: • the angle subtended by an arc of a circle at the centre is double the angle subtended by the arc at any point on the remaining part of the circumference • angles in the same segment are equal • the arcs are proportional to their corresponding angles at the circumference • the angle in a semi-circle is a right angle • if the angle at the circumference is a right angle, then the chord that subtends the angle is a diameter
	10.3 understand the properties of a cyclic quadrilateral		The properties of a cyclic quadrilateral include: • the opposite angles of a cyclic quadrilateral are supplementary • an exterior angle of a cyclic quadrilateral equals its interior opposite angle

Learning Unit	Learning Objective	Time	Remarks
	10.4 <u>understand the tests for concyclic points and cyclic quadrilaterals</u>		The tests for concyclic points and cyclic quadrilaterals include: • if A and D are two points on the same side of the line BC and $\angle BAC = \angle BDC$, then A, B, C and D are concyclic • if a pair of opposite angles of a quadrilateral are supplementary, then the quadrilateral is cyclic • if the exterior angle of a quadrilateral equals its interior opposite angle, then the quadrilateral is cyclic

Learning Unit	Learning Objective	Time	Remarks
	10.5 understand the properties of tangents to a circle and angles in the alternate segments		The properties include: • a tangent to a circle is perpendicular to the radius through the point of contact • the straight line perpendicular to a radius of a circle at its external extremity is a tangent to the circle • the perpendicular to a tangent at its point of contact passes through the centre of the circle • if two tangents are drawn to a circle from an external point, then: - the distances from the external point to the points of contact are equal - the tangents subtend equal angles at the centre - the straight line joining the centre to the external point bisects the angle between the tangents

Learning Unit	Learning Objective	Time	Remarks
	10.6 <u>use the basic properties of circles to perform simple geometric proofs</u>		• if a straight line is tangent to a circle, then the tangent-chord angle is equal to the angle in the alternate segment • if a straight line passes through an end point of a chord of a circle so that the angle it makes with the chord is equal to the angle in the alternate segment, then the straight line touches the circle

Learning Unit	Learning Objective	Time	Remarks
11. Locus	11.1 understand the concept of loci	7	
	11.2 describe and sketch the locus of points satisfying given conditions		The conditions include: • maintaining a fixed distance from a fixed point • maintaining an equal distance from two given points • maintaining a fixed distance from a line • maintaining a fixed distance from a line segment • maintaining an equal distance from two parallel lines • maintaining an equal distance from two intersecting lines
	11.3 describe the locus of points with algebraic equations		Students are expected to find the equations of simple loci, which include equations of straight lines, circles and parabolas (in the form of $y = ax^2 + bx + c$).

Learning Unit	Learning Objective	Time	Remarks
12. Equations of straight lines and circles	12.1 understand the equation of a straight line	14	Students are expected to find the equation of a straight line from given conditions such as: • the coordinates of any two points on the straight line • the slope of the straight line and the coordinates of a point on it • the slope and the *y*-intercept of the straight line Students are expected to describe the features of a straight line from its equation. The features include: • the slope • the intercepts with the axes • whether it passes through a given point The normal form is **not** required.

Learning Unit	Learning Objective	Time	Remarks
	12.2 understand the possible intersection of two straight lines		Students are expected to determine the number of intersection points of two straight lines from their equations. Note: Solving simultaneous linear equations in two unknowns is a learning objective at Key Stage 3.
	12.3 understand the equation of a circle		Students are expected to find the equation of a circle from given conditions such as: • the coordinates of the centre and the radius of the circle • the coordinates of any three points on the circle Students are expected to describe the features of a circle from its equation. The features include: • the centre • the radius • whether a given point lies inside, outside or on the circle

Learning Unit	Learning Objective	Time	Remarks
	12.4 find the coordinates of the intersections of a straight line and a circle and understand the possible intersection of a straight line and a circle		Finding the equations of tangents to a circle is required.
13. More about trigonometry	13.1 understand the functions sine, cosine and tangent, and their graphs and properties, including maximum and minimum values and periodicity 13.2 solve the trigonometric equations $a \sin \theta = b$, $a \cos \theta = b$, $a \tan \theta = b$ (solutions in the interval from 0° to 360°) and other trigonometric equations (solutions in the interval from 0° to 360°) 13.3 understand the formula $\frac{1}{2} ab \sin C$ for areas of triangles 13.4 understand the sine and cosine formulae 13.5 understand Heron's formula	21	Simplification of expressions involving sine, cosine and tangent of $-\theta$, $90° \pm \theta$, $180° \pm \theta$, … , etc. is required. Equations that can be transformed into quadratic equations are required only in the Non-foundation Topics and tackled in Learning Objective 5.3.

Learning Unit	Learning Objective	Time	Remarks
	13.6 <u>use the above formulae to solve 2-dimensional and 3-dimensional problems</u>		The "above formulae" refer to those mentioned in Learning Objectives 13.3 – 13.5. 3-dimensional problems include finding the angle between two lines, the angle between a line and a plane, the angle between two planes, the distance between a point and a line, and the distance between a point and a plane. Note: Exploring the properties of simple 3-D figures is a learning objective at Key Stage 3.
Data Handling Strand			
14. Permutation and combination	14.1 <u>understand the addition rule and multiplication rule in the counting principle</u>	11	
	14.2 <u>understand the concept and notation of permutation</u>		Notations such as "P_r^n", "${}_nP_r$", "${}^nP_r$", etc. can be used.

Learning Unit	Learning Objective	Time	Remarks
	14.3 solve problems on the permutation of distinct objects without repetition 14.4 understand the concept and notation of combination 14.5 solve problems on the combination of distinct objects without repetition		Problems such as "permutation of objects in which three particular objects are put next to each other" are required. Circular permutation is **not** required. Notations such as "C_r^n", "${}_nC_r$", "${}^nC_r$", "$\binom{n}{r}$", etc. can be used.
15. More about probability	15.1 recognise the notation of set language including union, intersection and complement 15.2 understand the addition law of probability and the concepts of mutually exclusive events and complementary events 15.3 understand the multiplication law of probability and the concept of independent events	10	The concept of Venn Diagram is required. The addition law of probability refers to "$P(A \cup B) = P(A) + P(B) - P(A \cap B)$". The multiplication law of probability refers to "$P(A \cap B) = P(A) \times P(B)$, where A and B are independent events".

Learning Unit	Learning Objective	Time	Remarks
	15.4 <u>recognise the concept and notation of conditional probability</u> 15.5 <u>use permutation and combination to solve problems relating to probability</u>		The rule "$P(A \cap B) = P(A) \times P(B \mid A)$" is required. Bayes' Theorem is **not** required.
16. Measures of dispersion	16.1 understand the concept of dispersion 16.2 understand the concepts of range and inter-quartile range 16.3 construct and interpret the box-and-whisker diagram and use it to compare the distributions of different sets of data 16.4 understand the concept of standard deviation for both grouped and ungrouped data sets 16.5 compare the dispersions of different sets of data using appropriate measures	14	A box-and-whisker diagram can also be called a "boxplot". The term "variance" should be introduced. Students are required to understand the following formula for standard deviation: $\sigma = \sqrt{\frac{(x_1 - \mu)^2 + \ldots + (x_N - \mu)^2}{N}}$.

Learning Unit	Learning Objective	Time	Remarks
	16.6 understand the applications of standard deviation to real-life problems involving standard scores and the normal distribution 16.7 explore the effect of the following operations on the dispersion of the data: (i) adding an item to the set of data (ii) removing an item from the set of data (iii) adding a common constant to each item of the set of data (iv) multiplying each item of the set of data by a common constant		
17. Uses and abuses of statistics	17.1 recognise different techniques in survey sampling and the basic principles of questionnaire design 17.2 discuss and recognise the uses and abuses of statistical methods in various daily-life activities or investigations	8	The concepts of “populations” and “samples” should be introduced. Probability sampling and non-probability sampling should be introduced. Students should recognise that, in constructing questionnaires, factors such as the types, wording and ordering of questions and response options influence their validity and reliability.

Learning Unit	Learning Objective	Time	Remarks
	17.3 assess statistical investigations presented in different sources such as news media, research reports, etc.		
Further Learning Unit			
18. Further applications	Solve more sophisticated real-life and mathematical problems that may require students to search the information for clues, to explore different strategies, or to integrate various parts of mathematics which they have learnt in different areas The main focuses are: (a) to explore and solve more sophisticated real-life problems (b) to appreciate the connections between different areas of mathematics	20	Examples: • solve simple financial problems in areas such as taxation and instalment payment • analyse and interpret data collected in surveys • explore and interpret graphs relating to real-life situations • explore Ptolemy's Theorem and its applications • model the relation between two sets of data which show a strong linear correlation and explore how to reduce simple non-linear relations such as $y = m\sqrt{x} + c$ and $y = k\,a^x$ to linear relations • explore the relation between the Fibonacci sequence and the Golden

Learning Unit	Learning Objective	Time	Remarks
			Ratio • appreciate the applications of cryptography • explore the Ceva's Theorem and its applications • investigate the causes and effects of the three crises in mathematics • analyse mathematical games (e.g. explore the general solution of the water puzzle)
19. Inquiry and investigation	Through various learning activities, discover and construct knowledge, further improve the ability to inquire, communicate, reason and conceptualise mathematical concepts	20	This is **not** an independent and isolated learning unit. The time is allocated for students to engage in learning activities from different learning units.

Grand total: 270 hours

2.6 Extended Part

The Extended Part is designed for students who need more mathematical knowledge and skills for their future studies and careers, and for those whose interests and maturity have been developed to a level that enables them to benefit from further mathematical study in different areas. The Extended Part aims at extending students' mathematics horizon beyond the Compulsory Part. Students have to handle more complicated problems in the Extended Part than in the Compulsory Part.

Two modules are offered as choices for students in the Extended Part. They are Module 1 (Calculus and Statistics) and Module 2 (Algebra and Calculus). Students are allowed to take at most one of the two modules.

Module 1 (Calculus and Statistics) is intended to cater for those students who will be involved in disciplines or careers which demand a wider scope and deeper understanding of mathematics, and for those who would like to learn more mathematical applications at the senior secondary level. It aims to:

- provide students with skills and concepts beyond the Compulsory Part;
- emphasise applications rather than mathematical rigour with a view to widening students' perspectives on mathematics; and
- provide students with intuitive concepts of calculus and statistics, related basic skills and useful tools for their future studies and careers.

Module 2 (Algebra and Calculus) is designed to suit the needs of students who will be involved in mathematics-related fields and careers, and those who would like to learn more in-depth mathematics at the senior secondary level. It aims to:

- provide students with skills and concepts beyond the Compulsory Part;
- emphasise understanding of mathematics for further progress in mathematically inclined disciplines; and
- provide students with a concrete foundation in algebra and calculus for their future studies and careers.

2.6.1 Organisation of Module 1 and Module 2

The organization of Module 1 (Calculus and Statistics) and Module 2 (Algebra and Calculus) is different from that of the Compulsory Part. The contents of these modules are usually interwoven. Instead of categorising the contents of the modules into strands as in the Compulsory Part, they are classified into different areas. Both modules have learning targets to set out the aims and directions for learning and teaching in the areas involved in the modules. The two modules are also organised into a hierarchy from Learning Targets to specific Learning Objectives.

For Module 1 (Calculus and Statistics), the three areas are Foundation Knowledge, Calculus and Statistics. For Module 2 (Algebra and Calculus), the three areas are Foundation Knowledge, Algebra and Calculus. In addition, the Further Learning Unit, independent from any of the above three areas in each module, is designed to enhance students' ability to inquire, communicate, reason and conceptualise mathematical concepts.

2.6.2 Learning Targets of Module 1 and Module 2

The learning targets of Module 1 (Calculus and Statistics) and Module 2 (Algebra and Calculus) are provided in the following tables:

Learning Targets of Module 1 (Calculus and Statistics)

Foundation Knowledge	Calculus	Statistics
Students are expected to:		
• apply binomial expansion for the study of probability and statistics; • model, graph and apply exponential functions and logarithmic functions to solve problems; and • understand the relationships between exponential and logarithmic functions and hence apply the two functions to solve real-life problems.	• recognise the concept of limits as the basis of differential and integral calculus; • understand the idea of differentiation and integration through consideration of concrete phenomena; and • find the derivatives, indefinite integrals and definite integrals of simple functions.	• understand the concepts of probability, random variables, and discrete and continuous probability distributions; • understand the fundamental ideas of statistical reasoning based on the Binomial, Poisson, Geometric and Normal Distributions; • use statistical ways of observing and thinking, and then make inferences; and • develop the ability to think mathematically about uncertainty and then apply such knowledge and skills to solve problems.

Learning Targets of Module 2 (Algebra and Calculus)

Foundation Knowledge	Algebra	Calculus
Students are expected to:		
• rationalise surd expressions; • understand the principle of mathematical induction; • expand binomials using the Binomial Theorem; • understand simple trigonometric functions and their graphs; • understand important trigonometric identities and formulae involving compound angles; and • understand the number *e*.	• understand the concepts, operations and properties of matrices and the inverses of square matrices up to order 3; • solve systems of linear equations; • understand the concept, operations and properties of vectors; and • apply the knowledge of vectors to solve problems in 2-dimensional space and 3-dimensional space.	• understand the concept of limits as the basis of differential and integral calculus; • understand the concepts and properties of derivatives, indefinite integrals and definite integrals of functions; • find the derivatives, indefinite integrals and definite integrals of simple functions; • find the second derivatives of functions; and • apply the knowledge of differentiation and integration to solve real-life problems.

2.6.3 Learning Objectives of Module 1 and Module 2

The time allocation for the Compulsory Part plus either one of the modules is 15% of the total lesson time (approximately 405 hours). To aid teachers in planning school-based curricula, a suggested lesson time in hours is provided against each learning unit.

The proposed learning objectives of the two modules are provided in the following tables:

Learning Objectives of Module 1 (Calculus and Statistics)

Notes:

1. *Learning units are grouped under three areas ("Foundation Knowledge", "Calculus" and "Statistics") and a Further Learning Unit.*
2. *Related learning objectives are grouped under the same learning unit.*
3. *The notes in the "Remarks" column of the table may be considered as supplementary information about the learning objectives.*
4. *To aid teachers in judging how far to take a given topic, a suggested lesson time in hours is given against each learning unit. However, the lesson time assigned is for their reference only. Teachers may adjust the lesson time to meet their individual needs.*

Learning Unit	Learning Objective	Time	Remarks
Foundation Knowledge Area			
1. Binomial expansion	1.1 recognise the expansion of $(a+b)^n$, where n is a positive integer	3	The use of the summation notation ($\sum$) should be introduced. The following contents are **not** required: • expansion of trinomials • the greatest coefficient, the greatest term and the properties of binomial coefficients • applications to numerical approximation

Learning Unit	Learning Objective	Time	Remarks
2. Exponential and logarithmic functions	2.1 recognise the definition of the number *e* and the exponential series $$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$$	7	
	2.2 recognise exponential functions and logarithmic functions		The following functions are required: • $y = e^x$ • $y = \ln x$
	2.3 use exponential functions and logarithmic functions to solve problems		Students are expected to know how to solve problems including those related to compound interest, population growth and radioactive decay.

Learning Unit	Learning Objective	Time	Remarks
	2.4 transform $y = kx^n$ and $y = ka^x$ to linear relations, where *a*, *n* and *k* are real numbers, $a > 0$ and $a \neq 1$		When experimental values of *x* and *y* are given, students can plot the graph of the corresponding linear relation from which they can determine the values of the unknown constants by considering its slope and intercept.
	Subtotal in hours	10	
Calculus Area			
Differentiation and Its Applications			
3. Derivative of a function	3.1 recognise the intuitive concept of the limit of a function	6	Students should be able to distinguish "continuous functions" and "discontinuous functions" from their graphs. Theorems on the limits of sum, difference, product, quotient, scalar multiplication of functions and the limits of composite functions should be stated without proof.

Learning Unit	Learning Objective	Time	Remarks
	3.2 find the limits of algebraic functions, exponential functions and logarithmic functions		The following types of algebraic functions are required: • polynomial functions • rational functions • power functions x^{α} • functions derived from the above ones through addition, subtraction, multiplication, division and composition, for example, $\sqrt{x^2+1}$
	3.3 recognise the concept of the derivative of a function from first principles		Students are **not** required to find the derivatives of functions from first principles. Notations including y', $f'(x)$ and $\frac{dy}{dx}$ should be introduced.
	3.4 recognise the slope of the tangent of the curve $y = f(x)$ at a point $x = x_0$		Notations including $f'(x_0)$ and $\left.\frac{dy}{dx}\right\|_{x=x_0}$ should be introduced.

Learning Unit	Learning Objective	Time	Remarks
4. Differentiation of a function	4.1 understand the addition rule, product rule, quotient rule and chain rule of differentiation	10	The following rules are required: • $\frac{d}{dx}(u+v)=\frac{du}{dx}+\frac{dv}{dx}$ • $\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$ • $\frac{d}{dx}(\frac{u}{v})=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$ • $\frac{dy}{dx}=\frac{dy}{du}\cdot\frac{du}{dx}$ • $\frac{dx}{dy}=\frac{1}{\frac{dy}{dx}}$

Learning Unit	Learning Objective	Time	Remarks
	4.2 find the derivatives of algebraic functions, exponential functions and logarithmic functions		The following formulae are required: • $(C)' = 0$ • $(x^n)' = nx^{n-1}$ • $(e^x)' = e^x$ • $(\ln x)' = \frac{1}{x}$ • $(\log_a x)' = \frac{1}{x \ln a}$ • $(a^x)' = a^x \ln a$ Implicit differentiation is **not** required. Logarithmic differentiation is required.

Learning Unit	Learning Objective	Time	Remarks
5. Second derivative	5.1 recognise the concept of the second derivative of a function 5.2 find the second derivative of an explicit function	2	Notations including y'', $f''(x)$ and $\frac{d^2y}{dx^2}$ should be introduced. Third and higher order derivatives are **not** required.
6. Applications of differentiation	6.1 use differentiation to solve problems involving tangents, rates of change, maxima and minima	9	Local and global extrema are required.
	Subtotal in hours	27	
Integration and Its Applications			
7. Indefinite integrals and their applications	7.1 recognise the concept of indefinite integration	10	Indefinite integration as the reverse process of differentiation should be introduced.

Learning Unit	Learning Objective	Time	Remarks
	7.2 understand the basic properties of indefinite integrals and basic integration formulae		The notation $\int f(x)\,dx$ should be introduced. The following properties are required: • $\int k\,f(x)dx = k\int f(x)\,dx$ • $\int [f(x) \pm g(x)]\,dx = \int f(x)\,dx \pm \int g(x)\,dx$ The following formulae are required and the meaning of the constant of integration C should be explained: • $\int k\,dx = kx + C$ • $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$, where $n \neq -1$ • $\int \frac{1}{x}\,dx = \ln\lvert x \rvert + C$ • $\int e^x\,dx = e^x + C$

Learning Unit	Learning Objective	Time	Remarks
	7.3 use basic integration formulae to find the indefinite integrals of algebraic functions and exponential functions 7.4 use integration by substitution to find indefinite integrals 7.5 use indefinite integration to solve problems		Integration by parts is **not** required.
8. Definite integrals and their applications	8.1 recognise the concept of definite integration	15	The definition of the definite integral as the limit of a sum of the areas of rectangles under a curve should be introduced. The notation $\int_a^b f(x)\,dx$ should be introduced. The knowledge of dummy variables, i.e. $\int_a^b f(x)\,dx = \int_a^b f(t)\,dt$ is required.

Learning Unit	Learning Objective	Time	Remarks
	8.2 recognise the Fundamental Theorem of Calculus and understand the properties of definite integrals		The Fundamental Theorem of Calculus refers to $\int_a^b f(x)\,dx = F(b) - F(a)$, where $\frac{d}{dx}F(x) = f(x)$. The following properties are required: • $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$ • $\int_a^a f(x)\,dx = 0$ • $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$ • $\int_a^b k\,f(x)\,dx = k\int_a^b f(x)\,dx$ • $\int_a^b [f(x) \pm g(x)]\,dx$ $= \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$

Learning Unit	Learning Objective	Time	Remarks
	8.3 find the definite integrals of algebraic functions and exponential functions 8.4 use integration by substitution to find definite integrals 8.5 use definite integration to find the areas of plane figures 8.6 use definite integration to solve problems		
9. Approximation of definite integrals using the trapezoidal rule	9.1 understand the trapezoidal rule and use it to estimate the values of definite integrals	4	Error estimation is **not** required.
	Subtotal in hours	29	
Statistics Area			
Further Probability			
10. Conditional probability and independence	10.1 understand the concepts of conditional probability and independent events 10.2 use the laws $P(A \cap B) = P(A)\,P(B \mid A)$ and $P(D \mid C) = P(D)$ for independent events C and D to solve problems	3	

Learning Unit	Learning Objective	Time	Remarks
11. Bayes' theorem	11.1 use Bayes' theorem to solve simple problems	4	
	Subtotal in hours	7	
Binomial, Geometric and Poisson Distributions and Their Applications			
12. Discrete random variables	12.1 recognise the concept of a discrete random variable	1	
13. Probability distribution, expectation and variance	13.1 recognise the concept of discrete probability distribution and its representation in the form of tables, graphs and mathematical formulae 13.2 recognise the concepts of expectation $E(X)$ and variance $\mathrm{Var}(X)$ and use them to solve simple problems 13.3 use the formulae $E(aX+b)=aE(X)+b$ and $\mathrm{Var}(aX+b)=a^2\,\mathrm{Var}(X)$ to solve simple problems	5	
14. Binomial distribution	14.1 recognise the concept and properties of the binomial distribution	5	Bernoulli distribution should be introduced. The mean and variance of the binomial distribution should be introduced (proofs are **not** required).

Learning Unit	Learning Objective	Time	Remarks
	14.2 calculate probabilities involving the binomial distribution		Use of the binomial distribution table is **not** required.
15. Geometric distribution	15.1 recognise the concept and properties of the geometric distribution 15.2 calculate probabilities involving the geometric distribution	4	The mean and variance of geometric distribution should be introduced (proofs are **not** required).
16. Poisson distribution	16.1 recognise the concept and properties of the Poisson distribution 16.2 calculate probabilities involving the Poisson distribution	4	The mean and variance of Poisson distribution should be introduced (proofs are **not** required). Use of the Poisson distribution table is **not** required.
17. Applications of binomial, geometric and Poisson distributions	17.1 use binomial, geometric and Poisson distributions to solve problems	5	
	Subtotal in hours	24	

Learning Unit	Learning Objective	Time	Remarks
Normal Distribution and Its Applications			
18. Basic definition and properties	18.1 recognise the concepts of continuous random variables and continuous probability distributions, with reference to the normal distribution 18.2 recognise the concept and properties of the normal distribution	3	Derivations of the mean and variance of the normal distribution are **not** required. The formulae written in Learning Objective 13.3 are also applicable to continuous random variables. Properties of the normal distribution include: • the curve is bell-shaped and symmetrical about the mean • the mean, mode and median are equal • the dispersion can be determined by the value of σ • the area under the curve is 1

Learning Unit	Learning Objective	Time	Remarks
19. Standardisation of a normal variable and use of the standard normal table	19.1 standardise a normal variable and use the standard normal table to find probabilities involving the normal distribution	2	
20. Applications of the normal distribution	20.1 find the values of $P(X > x_1)$, $P(X < x_2)$, $P(x_1 < X < x_2)$ and related probabilities, given the values of x_1, x_2, μ and σ, where $X \sim N(\mu, \sigma^2)$ 20.2 find the values of x, given the values of $P(X > x)$, $P(X < x)$, $P(a < X < x)$, $P(x < X < b)$ or a related probability, where $X \sim N(\mu, \sigma^2)$ 20.3 use the normal distribution to solve problems	7	
	Subtotal in hours	12	
Point and Interval Estimation			
21. Sampling distribution and point estimates	21.1 recognise the concepts of sample statistics and population parameters	7	

Learning Unit	Learning Objective	Time	Remarks
	21.2 recognise the sampling distribution of the sample mean from a random sample of size *n*		If the population mean is μ and population variance is σ^2, then the mean of the sample mean is μ and the variance of the sample mean is $\frac{\sigma^2}{n}$.
	21.3 recognise the concept of point estimates including the sample mean, sample variance and sample proportion		The concept of "estimator" should be introduced. If the population mean is μ and the population size is *N*, then the population variance is $\sigma^2 = \frac{\sum_{i=1}^{N}(x_i - \mu)^2}{N}$. If the sample mean is $\bar{x}$ and the sample size is *n*, then the sample variance is $s^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}$. Recognising the concept of "unbiased estimator" is required.
	21.4 recognise Central Limit Theorem		

Learning Unit	Learning Objective	Time	Remarks
22. Confidence interval for a population mean	22.1 recognise the concept of confidence interval 22.2 find the confidence interval for a population mean	6	• a 100(1 − α)% confidence interval for the mean μ of a normal population with known variance σ^2 is given by $(\bar{x} - z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}},\ \bar{x} + z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}})$ • when the sample size n is sufficiently large, a 100(1 − α)% confidence interval for the mean μ of a population with unknown variance is given by $(\bar{x} - z_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}},\ \bar{x} + z_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}})$, where s is the sample standard deviation
23. Confidence interval for a population proportion	23.1 find an approximate confidence interval for a population proportion	3	For a random sample of size n, where n is sufficiently large, drawn from a Bernoulli distribution, a 100(1 − α)% confidence interval for the population proportion p is given by $(\hat{p} - z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}},\ \hat{p} + z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}})$,

Learning Unit	Learning Objective	Time	Remarks
			where $\hat{p}$ is an unbiased estimator of the population proportion.
	Subtotal in hours	16	
Further Learning Unit			
24. Inquiry and investigation	Through various learning activities, discover and construct knowledge, further improve the ability to inquire, communicate, reason and conceptualise mathematical concepts	10	This is **not** an independent and isolated learning unit. The time is allocated for students to engage in learning activities from different learning units.
	Subtotal in hours	10	

Grand total: 135 hours

Learning Objectives of Module 2 (Algebra and Calculus)

Notes:

1. *Learning units are grouped under three areas ("Foundation Knowledge", "Algebra" and "Calculus") and a Further Learning Unit.*
2. *Related learning objectives are grouped under the same learning unit.*
3. *The notes in the "Remarks" column of the table may be considered as supplementary information about the learning objectives.*
4. *To aid teachers in judging how far to take a given topic, a suggested lesson time in hours is given against each learning unit. However, the lesson time assigned is for their reference only. Teachers may adjust the lesson time to meet their individual needs.*

Learning Unit	Learning Objective	Time	Remarks
Foundation Knowledge Area			
1 Surds	1.1 rationalise the denominators of expressions of the form $\frac{k}{\sqrt{a} \pm \sqrt{b}}$	1.5	This topic can be introduced when teaching limits and differentiation.

Learning Unit	Learning Objective	Time	Remarks
2. Mathematical induction	2.1 understand the principle of mathematical induction	5	Only the First Principle of Mathematical Induction is required. Applications to proving propositions related to the summation of a finite sequence and divisibility are included. Proving propositions involving inequalities is **not** required.
3. Binomial Theorem	3.1 expand binomials with positive integral indices using the Binomial Theorem	3	Proving the Binomial Theorem is required. The use of the summation notation ($\sum$) should be introduced. The following contents are **not** required: • expansion of trinomials • the greatest coefficient, the greatest term and the properties of binomial coefficients • applications to numerical approximation

Learning Unit	Learning Objective	Time	Remarks
4. More about trigonometric functions	4.1 understand the concept of radian measure 4.2 find arc lengths and areas of sectors through radian measure 4.3 understand the functions cosecant, secant and cotangent and their graphs 4.4 understand the identities $1 + \tan^2 \theta = \sec^2 \theta$ and $1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$ 4.5 understand compound angle formulae and double angle formulae for the functions sine, cosine and tangent, and product-to-sum and sum-to-product formulae for the functions sine and cosine	11	Simplifying trigonometric expressions by identities is required. The following formulae are required: • $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$ • $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$ • $\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$ • $\sin 2A = 2 \sin A \cos A$ • $\cos 2A = \cos^2 A - \sin^2 A$ $= 1 - 2 \sin^2 A = 2 \cos^2 A - 1$ • $\tan 2A = \dfrac{2 \tan A}{1 - \tan^2 A}$ • $\sin^2 A = \dfrac{1}{2}(1 - \cos 2A)$

Learning Unit	Learning Objective	Time	Remarks
			• $\cos^2 A = \frac{1}{2}(1 + \cos 2A)$ • $2 \sin A \cos B = \sin(A + B) + \sin(A - B)$ • $2 \cos A \cos B = \cos(A + B) + \cos(A - B)$ • $2 \sin A \sin B = \cos(A - B) - \cos(A + B)$ • $\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$ • $\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$ • $\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$ • $\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$ "Subsidiary angle form" is **not** required. $\sin^2 A = \frac{1}{2}(1-\cos 2A)$ and $\cos^2 A = \frac{1}{2}(1+\cos 2A)$ can be considered as formulae derived from the double angle formulae.

Learning Unit	Learning Objective	Time	Remarks
5. Introduction to the number *e*	5.1 recognise the definitions and notations of the number *e* and the natural logarithm	1.5	Two approaches for the introduction to *e* can be considered: • $e = \lim_{n\to\infty}(1+\frac{1}{n})^n$ (proving the existence of this limit is **not** required) • $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$ This section can be introduced when teaching Learning Objective 6.1.
	Subtotal in hours	22	

Learning Unit	Learning Objective	Time	Remarks
Calculus Area			
Limits and Differentiation			
6. Limits	6.1 understand the intuitive concept of the limit of a function	5	Students should be able to distinguish "continuous functions" and "discontinuous functions" from their graphs. The absolute value function $\lvert x \rvert$, signum function $\operatorname{sgn}(x)$, ceiling function $\lceil x \rceil$ and floor function $\lfloor x \rfloor$ are examples of continuous functions and discontinuous functions. The theorem on the limits of sum, difference, product, quotient, scalar multiple and composite functions should be introduced but the proofs are **not** required.

Learning Unit	Learning Objective	Time	Remarks
	6.2 find the limit of a function		The following formulae are required: • $\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$ • $\lim_{x \to 0} \frac{e^x - 1}{x} = 1$ Finding the limit of a rational function at infinity is required.
7. Differentiation	7.1 understand the concept of the derivative of a function 7.2 understand the addition rule, product rule, quotient rule and chain rule of differentiation	14	Students should be able to find the derivatives of elementary functions, including C, x^n (n is a positive integer), $\sqrt{x}$, $\sin x$, $\cos x$, e^x, $\ln x$ from first principles. Notations including y', $f'(x)$ and $\frac{dy}{dx}$ should be introduced. Testing differentiability of functions is **not** required. The following rules are required:

Learning Unit	Learning Objective	Time	Remarks
			• $\frac{d}{dx}(u+v)=\frac{du}{dx}+\frac{dv}{dx}$ • $\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$ • $\frac{d}{dx}(\frac{u}{v})=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$ • $\frac{dy}{dx}=\frac{dy}{du}\cdot\frac{du}{dx}$
	7.3 find the derivatives of functions involving algebraic functions, trigonometric functions, exponential functions and logarithmic functions		The following formulae are required: • $(C)' = 0$ • $(x^n)' = n\,x^{n-1}$ • $(\sin x)' = \cos x$ • $(\cos x)' = -\sin x$ • $(\tan x)' = \sec^2 x$ • $(\cot x)' = -\mathrm{cosec}^2 x$ • $(\sec x)' = \sec x \tan x$ • $(\mathrm{cosec}\, x)' = -\mathrm{cosec}\, x \cot x$ • $(e^x)' = e^x$ • $(\ln x)' = \frac{1}{x}$

Learning Unit	Learning Objective	Time	Remarks
			The following types of algebraic functions are required: • polynomial functions • rational functions • power functions x^{α} • functions formed from the above functions through addition, subtraction, multiplication, division and composition, for example $\sqrt{x^2+1}$
	7.4 find derivatives by implicit differentiation		Logarithmic differentiation is required.
	7.5 find the second derivative of an explicit function		Notations including y'', $f''(x)$ and $\frac{d^2y}{dx^2}$ should be introduced. Third and higher order derivatives are **not** required.
8. Applications of differentiation	8.1 find the equations of tangents and normals to a curve	14	
	8.2 find maxima and minima		Local and global extrema are required.

Learning Unit	Learning Objective	Time	Remarks
	8.3 sketch curves of polynomial functions and rational functions 8.4 solve the problems relating to rate of change, maximum and minimum		The following points are noteworthy in curve sketching: • symmetry of the curve • limitations on the values of x and y • intercepts with the axes • maximum and minimum points • points of inflexion • vertical, horizontal and oblique asymptotes to the curve Students may deduce the equation of the oblique asymptote to the curve of a rational function by division.
	Subtotal in hours	33	

Learning Unit	Learning Objective	Time	Remarks
Integration			
9. Indefinite integration	9.1 recognise the concept of indefinite integration 9.2 understand the properties of indefinite integrals and use the integration formulae of algebraic functions, trigonometric functions and exponential functions to find indefinite integrals	16	Indefinite integration as the reverse process of differentiation should be introduced. The following formulae are required: • $\int k\,dx = kx + C$ • $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$, where $n \neq -1$ • $\int \frac{1}{x}\,dx = \ln\lvert x \rvert + C$ • $\int e^x\,dx = e^x + C$ • $\int \sin x\,dx = -\cos x + C$ • $\int \cos x\,dx = \sin x + C$ • $\int \sec^2 x\,dx = \tan x + C$ • $\int \operatorname{cosec}^2 x\,dx = -\cot x + C$ • $\int \sec x \tan x\,dx = \sec x + C$ • $\int \operatorname{cosec} x \cot x\,dx = -\operatorname{cosec} x + C$ For more complicated calculations, see Learning Objectives 9.4 to 9.6.

Learning Unit	Learning Objective	Time	Remarks
	9.3 understand the applications of indefinite integrals in real-life or mathematical contexts		Applications of indefinite integrals in some fields such as geometry and physics are required.
	9.4 use integration by substitution to find indefinite integrals		
	9.5 use trigonometric substitutions to find the indefinite integrals involving $\sqrt{a^2 - x^2}$, $\sqrt{x^2 - a^2}$ or $\sqrt{a^2 + x^2}$		Notations including $\sin^{-1} x$, $\cos^{-1} x$ and $\tan^{-1} x$ and their related principal values should be introduced.
	9.6 use integration by parts to find indefinite integrals		$\int \ln x \, dx$ can be used as an example to illustrate the method of integration by parts. The use of integration by parts is limited to at most two times in finding an integral.

Learning Unit	Learning Objective	Time	Remarks
10. Definite integration	10.1 recognise the concept of definite integration	11	The definition of the definite integral as the limit of a sum and finding a definite integral from the definition should be introduced. The use of dummy variables, including $\int_a^b f(x)\,dx = \int_a^b f(t)\,dt$, is required. Using definite integration to find the sum to infinity of a sequence is **not** required.
	10.2 understand the properties of definite integrals		The following properties are required: • $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$ • $\int_a^a f(x)\,dx = 0$ • $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$ • $\int_a^b k\,f(x)\,dx = k\int_a^b f(x)\,dx$ • $\int_a^b [f(x) \pm g(x)]\,dx$ $= \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$

Learning Unit	Learning Objective	Time	Remarks
	10.3 find definite integrals of algebraic functions, trigonometric functions and exponential functions		Fundamental Theorem of Calculus: $\int_a^b f(x)\,dx = F(b) - F(a)$, where $\frac{d}{dx}F(x) = f(x)$, should be introduced.
	10.4 use integration by substitution to find definite integrals		
	10.5 use integration by parts to find definite integrals		The use of integration by parts is limited to at most two times in finding an integral.
	10.6 understand the properties of the definite integrals of even, odd and periodic functions		The following properties are required: • $\int_{-a}^{a} f(x)\,dx = 0$ if f is odd • $\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx$ if f is even • $\int_0^{nT} f(x)\,dx = n\int_0^T f(x)\,dx$ if $f(x+T) = f(x)$, i.e. f is periodic

Learning Unit	Learning Objective	Time	Remarks
11. Applications of definite integration	11.1 understand the application of definite integrals in finding the area of a plane figure 11.2 understand the application of definite integrals in finding the volume of a solid of revolution about a coordinate axis or a line parallel to a coordinate axis	7	Both “disc method” and “shell method” are required. Finding the volume of a hollow solid is required.
	Subtotal in hours	34	
Algebra Area			
Matrices and Systems of Linear Equations			
12. Determinants	12.1 recognise the concept and properties of determinants of order 2 and order 3	3	The following properties are required: • $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$ • $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = -\begin{vmatrix} c_1 & b_1 & a_1 \\ c_2 & b_2 & a_2 \\ c_3 & b_3 & a_3 \end{vmatrix}$ • $\begin{vmatrix} a_1 & b_1 & 0 \\ a_2 & b_2 & 0 \\ a_3 & b_3 & 0 \end{vmatrix} = 0$

Learning Unit	Learning Objective	Time	Remarks
			• $\begin{vmatrix} a_1 & kb_1 & c_1 \\ a_2 & kb_2 & c_2 \\ a_3 & kb_3 & c_3 \end{vmatrix} = k\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ • $\begin{vmatrix} a_1 & b_1 & kb_1 \\ a_2 & b_2 & kb_2 \\ a_3 & b_3 & kb_3 \end{vmatrix} = 0$ • $\begin{vmatrix} a_1 + a_1' & b_1 & c_1 \\ a_2 + a_2' & b_2 & c_2 \\ a_3 + a_3' & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + \begin{vmatrix} a_1' & b_1 & c_1 \\ a_2' & b_2 & c_2 \\ a_3' & b_3 & c_3 \end{vmatrix}$ • $\begin{vmatrix} a_1 + kb_1 & b_1 & c_1 \\ a_2 + kb_2 & b_2 & c_2 \\ a_3 + kb_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ • $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1\begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - a_2\begin{vmatrix} b_1 & c_1 \\ b_3 & c_3 \end{vmatrix} + a_3\begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}$ Notations including $\lvert A \rvert$ and $\det(A)$ should be introduced.
13. Matrices	13.1 understand the concept, operations and properties of matrices	9	The addition, scalar multiplication and multiplication of matrices are required. The following properties are required: • $A + B = B + A$

Learning Unit	Learning Objective	Time	Remarks
			• $A + (B + C) = (A + B) + C$ • $(\lambda + \mu)A = \lambda A + \mu A$ • $\lambda(A + B) = \lambda A + \lambda B$ • $A(BC) = (AB)C$ • $A(B + C) = AB + AC$ • $(A + B)C = AC + BC$ • $(\lambda A)(\mu B) = (\lambda\mu)AB$ • $\lvert AB \rvert = \lvert A \rvert \lvert B \rvert$
	13.2 understand the concept, operations and properties of inverses of square matrices of order 2 and order 3		The following properties are required: • the inverse of A is unique • $(A^{-1})^{-1} = A$ • $(\lambda A)^{-1} = \lambda^{-1}A^{-1}$ • $(A^n)^{-1} = (A^{-1})^n$ • $(A^t)^{-1} = (A^{-1})^t$ • $\lvert A^{-1} \rvert = \lvert A \rvert^{-1}$ • $(AB)^{-1} = B^{-1}A^{-1}$ where A and B are invertible matrices and λ is a non-zero scalar.

Learning Unit	Learning Objective	Time	Remarks
14. Systems of linear equations	14.1 solve the systems of linear equations of order 2 and order 3 by Cramer's rule, inverse matrices and Gaussian elimination	6	The following theorem is required: • A system of homogeneous linear equations in three unknowns has nontrivial solutions if and only if the coefficient matrix is singular The wording "necessary and sufficient conditions" could be introduced to students.
	Subtotal in hours	18	
Vectors			
15. Introduction to vectors	15.1 understand the concepts of vectors and scalars	5	The concepts of magnitudes of vectors, zero vector and unit vectors are required. Students should recognise some common notations of vectors in printed form (including $\mathbf{a}$ and $\overrightarrow{AB}$) and in written form (including $\vec{a}$, $\overrightarrow{AB}$ and $\underline{a}$); and some notations for magnitude (including $\lvert \mathbf{a} \rvert$ and $\lvert \vec{a} \rvert$).

Learning Unit	Learning Objective	Time	Remarks
	15.2 understand the operations and properties of vectors		The addition, subtraction and scalar multiplication of vectors are required. The following properties are required: • $\mathbf{a} + \mathbf{b} = \mathbf{b} + \mathbf{a}$ • $\mathbf{a} + (\mathbf{b} + \mathbf{c}) = (\mathbf{a} + \mathbf{b}) + \mathbf{c}$ • $\mathbf{a} + \mathbf{0} = \mathbf{a}$ • $0\,\mathbf{a} = \mathbf{0}$ • $\lambda(\mu\mathbf{a}) = (\lambda\mu)\mathbf{a}$ • $(\lambda + \mu)\mathbf{a} = \lambda\mathbf{a} + \mu\mathbf{a}$ • $\lambda(\mathbf{a} + \mathbf{b}) = \lambda\mathbf{a} + \lambda\mathbf{b}$ • If $\alpha\mathbf{a} + \beta\mathbf{b} = \alpha_1\mathbf{a} + \beta_1\mathbf{b}$ ($\mathbf{a}$ and $\mathbf{b}$ are non-zero and are not parallel to each other), then $\alpha = \alpha_1$ and $\beta = \beta_1$
	15.3 understand the representation of a vector in the rectangular coordinate system		The following formulae are required: • $\lvert \overrightarrow{OP} \rvert = \sqrt{x^2 + y^2 + z^2}$ in $\mathbf{R}^3$ • $\sin\theta = \dfrac{y}{\sqrt{x^2 + y^2}}$ and

Learning Unit	Learning Objective	Time	Remarks
			$\cos\theta = \dfrac{x}{\sqrt{x^2+y^2}}$ in $\mathbf{R}^2$ The representation of vectors in the rectangular coordinate system can be used to discuss those properties listed in the Remarks against Learning Objective 15.2. The concept of direction cosines is **not** required.
16. Scalar product and vector product	16.1 understand the definition and properties of the scalar product (dot product) of vectors 16.2 understand the definition and properties of the vector product (cross product) of vectors in $\mathbf{R}^3$	5	The following properties are required: • $\mathbf{a}\cdot\mathbf{b} = \mathbf{b}\cdot\mathbf{a}$ • $\mathbf{a}\cdot(\lambda\mathbf{b}) = \lambda(\mathbf{a}\cdot\mathbf{b})$ • $\mathbf{a}\cdot(\mathbf{b}+\mathbf{c}) = \mathbf{a}\cdot\mathbf{b}+\mathbf{a}\cdot\mathbf{c}$ • $\mathbf{a}\cdot\mathbf{a} = \lvert\mathbf{a}\rvert^2 \geq 0$ • $\mathbf{a}\cdot\mathbf{a} = 0$ if and only if $\mathbf{a} = \mathbf{0}$ • $\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert \geq \lvert\mathbf{a}\cdot\mathbf{b}\rvert$ • $\lvert\mathbf{a}-\mathbf{b}\rvert^2 = \lvert\mathbf{a}\rvert^2 + \lvert\mathbf{b}\rvert^2 - 2(\mathbf{a}\cdot\mathbf{b})$ The following properties are required: • $\mathbf{a}\times\mathbf{a} = \mathbf{0}$

Learning Unit	Learning Objective	Time	Remarks
			• $\mathbf{b} \times \mathbf{a} = -(\mathbf{a} \times \mathbf{b})$ • $(\mathbf{a} + \mathbf{b}) \times \mathbf{c} = \mathbf{a} \times \mathbf{c} + \mathbf{b} \times \mathbf{c}$ • $\mathbf{a} \times (\mathbf{b} + \mathbf{c}) = \mathbf{a} \times \mathbf{b} + \mathbf{a} \times \mathbf{c}$ • $(\lambda\mathbf{a}) \times \mathbf{b} = \mathbf{a} \times (\lambda\mathbf{b}) = \lambda(\mathbf{a} \times \mathbf{b})$ • $\lvert \mathbf{a} \times \mathbf{b} \rvert^2 = \lvert \mathbf{a} \rvert^2 \lvert \mathbf{b} \rvert^2 - (\mathbf{a} \cdot \mathbf{b})^2$ The following properties of scalar triple products should be introduced: • $(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c} = \mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$ • $(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c} = (\mathbf{b} \times \mathbf{c}) \cdot \mathbf{a} = (\mathbf{c} \times \mathbf{a}) \cdot \mathbf{b}$
17. Applications of vectors	17.1 understand the applications of vectors	8	Division of a line segment, parallelism and orthogonality are required. Finding angles between two vectors, the projection of a vector onto another vector, the volume of a parallelepiped and the area of a triangle are required.
	Subtotal in hours	18	

Learning Unit	Learning Objective	Time	Remarks
Further Learning Unit			
18. Inquiry and investigation	Through various learning activities, discover and construct knowledge, further improve the ability to inquire, communicate, reason and conceptualise mathematical concepts	10	This is **not** an independent and isolated learning unit. The time is allocated for students to engage in learning activities from different learning units.
	Subtotal in hours	10	

Grand total: 135 hours